TEMPERATURE DISTRIBUTION IN A ROD WITH CONSIDERATION OF RADIATION AND VARIABILITY IN PHYSICAL CHARACTERISTICS

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We propose a method of calculating the temperature distribution for rods of variable constant cross section, with consideration of radiation and the variability of the physical characteristics. We present the results from an experimental verification of the derived solution.

Let us examine the transfer of heat in a metal rod heated at the base and situated in a medium with the temperature T_a, when the thermophysical characteristics are specified functions of temperature. The problem reduces to the solution of the nonlinear differential equation

$$F\frac{d}{dx}\left(\lambda\frac{dT}{dx}\right)=\alpha\chi\,(T-T_a)+\sigma\varepsilon\chi\left[\left(\frac{T}{100}\right)^4-\left(\frac{T_a}{100}\right)^4\right] \tag{1}$$

with the boundary conditions

$$T=T_0 \text{ when } x=0;\quad -\lambda\frac{dT}{dx}=\alpha_l(T-T_a) \text{ when } x=l. \tag{2}$$

In the last boundary condition we can neglect radiation, in the assumption of a relatively small magnitude for the temperature head at the end of the rod.

If a rod of varying cross section is made up of several rods of constant cross section, Eq. (1) retains its form for each stage. The boundary conditions (2) remain unchanged, and to these we add the condition of equality for the temperatures and heat flows in the plane of stage separation:

$$T_i=T_{i+1} \text{ when } x=l_i;\quad F_i\frac{dT_i}{dx}=F_{i+1}\frac{dT_{i+1}}{dx} \text{ when } x=l_i. \tag{3}$$

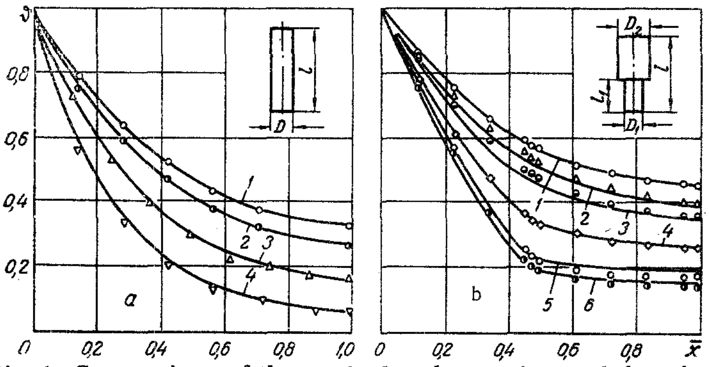

Fig. 1. Comparison of theoretical and experimental data for rods of constant cross section (a): 1) $l/D = 5$, $T_0 = 420$°K; 2) 5, 675; 3) 8, 675; 4) 17.5, 925; and for rods of varying cross section (b): 1) $F_1/F_2 = 0.7$, $T_0 = 580$; 2) 0.7, 890; 3) 0.5, 860; 4) 0.3, 1050; 5) 0.1, 480; 6) 0.1, 890°K.

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The possibility of a one-dimensional formulation of the problem when Bi ≪ 1 can be evaluated by means of the relationship for the temperature distribution in the lateral cross section of a cylindrical rod, i.e.,

$$\frac{T-T_a}{T_c-T_a}=1-\frac{1}{2}\,\mathrm{Bi}\left(\frac{r}{D}\right)^2. \tag{4}$$

For the rods under consideration we have Bi < 0.05, and the nonuniformity in the temperature distribution through the lateral cross section does not exceed 1%.

The coefficient of thermal conductivity as a function of temperature for many industrial alloys can be expressed [1] by the linear function

$$\lambda(T)=\lambda_0(1+cT). \tag{5}$$

To calculate the heat-transfer coefficient [2] for the case of laminar free convection we use the equation

$$\mathrm{Nu}_{ax}=0.60\,(\mathrm{Gr}_{ax}\,\mathrm{Pr}_a)^{0.25}\left(\frac{\mathrm{Pr}_a}{\mathrm{Pr}_r}\right)^{0.25}. \tag{6}$$

The emissivity for metals in which the radiated energy increases more rapidly than temperature to the fourth power may be regarded as proportional to the absolute temperature of these metals [3]. The total radiant energy is therefore assumed with sufficient accuracy for this problem to be proportional to the absolute temperature raised to the fifth power, i.e.,

$$a\chi\left[\left(\frac{T}{100}\right)^5-\left(\frac{T_a}{100}\right)^5\right]. \tag{7}$$

The values for the coefficient a for certain metals are given in [3].

An analytical solution for (1) with the boundary conditions (2) and (3) is possible for certain special examples of the problem relating the physical characteristics to the temperature and to the coordinate [4]. In the general case, Eq. (1) should be integrated numerically.

The figure shows the curves of the theoretical relationship between the dimensionless excess temperature $\vartheta = (T - T_a)/(T_0 - T_a)$ and the dimensionless coordinates $\bar{x} = x/l$. The results of the numerical calculations are compared with the experimental data. The measured temperature values are indicated by special notation.

The tests were carried out in a steady-state regime on vertical rods made of Kh18N9T steel (GOST 5632-61), heated from below, with natural convection in the air medium.

The temperature was measured with Chromel–Alumel thermocouples at 8-12 points through the height of the rod. The measurements were carried out both at the center and at the surface of the rod, at each of the measuring points. The measurement results were processed both for the temperature at the center and at the surface. The resulting values of the temperature head were averaged for comparison with the results from calculation in the one-dimensional approximation.

The temperature distribution in rods of constant cross section (see Fig. 1a) was studied over the range $420°K \le T_0 \le 925°K$ on cylindrical models whose diameters ranged from 20 to 70 mm, with $5 \le l/D \le 17.5$. The measurement results shown in Fig. 1b for a two-stage rod were found in the range $480°K \le T_0 \le 1050°K$ for $l_1/l = 0.462$ and an area relationship in the limits $0.1 \le F_1/F_2 \le 0.7$.

The results from the numerical integration of (1) are in good agreement with the experiment. The maximum divergence is found in the plane of stage separation and does not exceed 6% for rods of varying cross section, with $0.3 \le F_1/F_2 \le 0.7$. Somewhat greater deviations are found in the case of a low value for the temperature head (see Fig. 1b, curves 5 and 6), but in terms of absolute magnitude they do not exceed 10°C.

NOTATION

T is the absolute temperature;
x is the coordinate;

F	is the cross-sectional area;
χ	is the perimeter;
l	is the linear dimension;
r	is the instantaneous radius of the cylinder;
D	is the diameter;
λ	is the coefficient of thermal conductivity;
α	is the heat-transfer coefficient;
ε	is the emissivity;
σ	is the Stefan–Boltzmann constant;
$Nu_x = \alpha x/\lambda$	is the Nusselt number;
$Gr_x = g\beta x^3 \Delta t/\nu^2$	is the Grashof number;
$Pr = \nu/a$	is the Prandtl number;

Symbols

a	denotes the ambient medium;
l	denotes the end of the rod;
c	denotes the center of the rod.

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